

Three Known Extensions of $E = mc^2$

Two Valid (Gravity, Electromagnetism) + One Conceptual (Higgs)

$$\text{From } E = \gamma mc^2 \text{ to } E = \gamma (mc^2 + m\Phi)$$

$$\text{and } E = \sqrt{(\mathbf{p} - q\mathbf{A})^2 c^2 + m_0^2 c^4} + q\phi$$

$$\text{and } mc^2 = \mathbf{y} \cdot \mathbf{v}$$

Jan Klein, 19 April 2026, Hannover, Germany - ORCID: [0009-0002-2951-995X](https://orcid.org/0009-0002-2951-995X)

Contact: bix.pages.dev@gmail.com - Website: bix.pages.dev

Keywords: mass-energy equivalence, $E = mc^2$, general relativity, electromagnetism, Higgs mechanism, quantum field theory, weak field approximation, minimal coupling, Yukawa coupling, proton mass, gluon confinement, Pound-Rebka, GPS, LIGO, ATLAS, CMS, historical foundations, reductionism, structural realism, conventionalism

Abstract

Einstein's energy-momentum relation $E = \gamma mc^2$ describes a free particle in empty space. However, no real particle exists in isolation. Every particle interacts with fields. This paper reviews three known ways to extend $E = \gamma mc^2$ to include such interactions, based on established and experimentally confirmed physics.

Extension 1 - Gravity (Einstein, 1915):

$$E \approx \gamma(mc^2 + m\Phi), \quad \Phi = -GM/r$$

Extension 2 - Electromagnetism (Maxwell-Einstein, 1905-1915):

$$\text{Exact: } E = \sqrt{(\mathbf{p} - q\mathbf{A})^2 c^2 + m_0^2 c^4} + q\phi$$

Weak-field: $E \approx \gamma m_0 c^2 + q\phi$ - valid for weak static electric fields, confirmed by all electrodynamics and QED.

Extension 3 - Higgs Mechanism (Higgs, 1964; confirmed 2012):

$$mc^2 = \mathbf{y} \cdot \mathbf{v}, \quad v \approx 246 \text{ GeV}$$

The paper presents these three extensions in historical order, provides their exact and approximate forms, states their domains of validity, and summarizes key experimental confirmations. No new physics is proposed. The goal is to collect and clarify what is already known: $\mathbf{E} = \mathbf{m}\mathbf{c}^2$ can be extended in exactly three well-established ways - two direct (gravity, electromagnetism) and one conceptual (Higgs). This work is the conceptual launchpad to build a new extended energy equation.

1. Introduction: From Empty Space to Field Interactions

In 1905, Albert Einstein derived from his special theory of relativity the relation $\mathbf{E} = \gamma\mathbf{m}\mathbf{c}^2$ between the total energy of a free particle, its rest mass, and its velocity. For a particle at rest, this reduces to the most famous equation in physics: $\mathbf{E} = \mathbf{m}\mathbf{c}^2$. These equations assume empty space. No external fields act on the particle. The particle's energy depends only on its own mass and motion.

But no real particle exists in empty space. Every charged particle moves through electromagnetic fields. Every massive particle moves through gravitational fields. Every elementary fermion (electron, quark) couples to the Higgs field. The proton - the building block of ordinary matter - derives 99% of its mass not from the rest masses of its constituent quarks, but from the energy of the gluon field that confines them.

This raises a natural question: How does the energy equation change when we stop assuming empty space? The answer is not a single new formula. Different fields enter the energy equation in different ways, and some fields (like the strong nuclear field) cannot be written as a simple additive potential at all. However, three clear, well-established extensions exist. They come from different eras of physics, have different mathematical structures, and are confirmed by different experiments.

The three extensions are: Gravity (weak-field limit adds $\mathbf{m}\Phi$); Electromagnetism (minimal coupling yields $\mathbf{E} = \sqrt{(\mathbf{p} - \mathbf{q}\mathbf{A})^2\mathbf{c}^2 + \mathbf{m}_0^2\mathbf{c}^4} + \mathbf{q}\phi$); and the Higgs mechanism (mass itself is $\mathbf{y} \cdot \mathbf{v}/\mathbf{c}^2$, a conceptual rather than additive extension). This paper is a reference for physicists and the necessary foundation for any future unification or novel extension of the mass-energy relation.

2. Historical Foundations: From Galileo to Today

The following list presents the essential contributions that led to our understanding of energy, mass, and their relation to fields. Each entry directly enabled the next step.

1638 - Galilei

$$s = \frac{1}{2}gt^2, \text{ law of inertia}$$

First to understand inertia, later recognized as mass.

1687 - Newton

$$F = ma, \quad F = G \frac{m_1 m_2}{r^2}$$

Mass as inertia and source of gravity.

1687 - Leibniz

$$mv^2 \text{ (vis viva)}$$

Precursor to kinetic energy.

1800 - Rumford

Heat = motion

Energy conservation emerging.

1842 - Mayer

Energy conservation $E = \text{const}$

First to state energy conservation.

1847 - Helmholtz

$$\Delta E = 0$$

Energy conservation for mechanical, thermal, electrical, chemical processes.

1865 - Maxwell

$$u = \frac{1}{2}(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2)$$

Field energy, not only particles.

1881 - Thomson (J.J.)

$$m_{\text{em}} = \frac{4}{3} \frac{E_{\text{em}}}{c^2}$$

Electromagnetic contribution to mass.

1889 - Wien

$$m \approx m_0 + E_{\text{kin}}/c^2$$

Kinetic energy adds to mass.

1900 - Poincaré

$$E = mc^2 \text{ (qualitative)}$$

Electromagnetic energy has inertial mass.

1904 - Hasenöhrl

$$m = \frac{4}{3} \frac{E}{c^2} \text{ (cavity radiation)}$$

Closest precursor to Einstein.

1905 - Einstein

$$E = mc^2 \text{ (rest)}, \quad E = \gamma mc^2 \text{ (motion)}$$

Mass-energy equivalence.

1906 - Planck

$$m = E/c^2$$

Refined relativistic mass.

1908 - Minkowski

$$p^\mu = mu^\mu$$

Four-vector embedding of energy-momentum.

1911 - Bohr

$$E = h\nu$$

Quantized transitions, basis for nuclear tests.

1915 - Einstein

$$E \approx \gamma(mc^2 + m\Phi)$$

First valid extension (gravity).

1915-1940s - Various

$$E = \sqrt{(\mathbf{p} - q\mathbf{A})^2 c^2 + m_0^2 c^4} + q\phi$$

Second valid extension (electromagnetism).

1932 - Chadwick

$$m_n \approx 939.6 \text{ MeV}/c^2$$

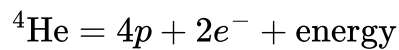
Neutron discovery.

1934 - Fermi

$$Q = (m_i - m_f)c^2$$

Mass defects confirm $E = \Delta mc^2$.

1939 - Bethe



Fusion in stars, confirmation of $E = mc^2$.

1955 - Bainbridge *et al.*

$$E = \Delta mc^2 \text{ (0.5\% precision)}$$

First direct experimental test.

1964 - Higgs

$$mc^2 = y \cdot v$$

Third (conceptual) extension - mass from field coupling.

1967 - Weinberg

Electroweak model

Unified electromagnetic and weak forces, predicted W/Z masses.

1971 - Hafele-Keating

$$\Delta t = \gamma \Delta t_0$$

Relativistic time dilation confirmed.

1983 - UA1/UA2

$$m_W \approx 80.4 \text{ GeV}/c^2, \quad m_Z \approx 91.2 \text{ GeV}/c^2$$

W and Z boson discovery, confirmed Higgs mechanism predictions.

1995 - CDF/D0

$$m_t \approx 173 \text{ GeV}/c^2$$

Top quark discovery, large Yukawa coupling consistent with Higgs.

2005 - Rainville *et al.*

$$E = \Delta mc^2 \quad (0.0004\% \text{ precision})$$

Most precise direct test of mass-energy equivalence.

2012 - ATLAS/CMS

$$m_H \approx 125 \text{ GeV}/c^2$$

Higgs boson discovery, field origin of mass.

2015-present - LIGO/Virgo

$$E = \Delta mc^2 \quad (\text{black hole mergers})$$

Gravitational waves carry mass-energy.

2022 - BASE (CERN)

$$m_p = E/c^2 \quad \text{with } 2 \times 10^{-11} \text{ precision}$$

Most precise proton mass via $E = mc^2$.

2026 - Jan Klein (this work)

Proposes a new extension of the energy equation

Building on these three foundations.

3. Extension 1: Gravity (Einstein, 1915)

For a particle of mass \mathbf{m} in a weak gravitational potential $\Phi = -GM/r$, the total energy to first order in Φ/c^2 is:

$$\mathbf{E} \approx \gamma \mathbf{m} c^2 \left(1 + \frac{\Phi}{c^2} \right) = \gamma (\mathbf{m} c^2 + \mathbf{m} \Phi)$$

Minor clarification: The exact expression from the Schwarzschild metric is $\mathbf{E} = \gamma \mathbf{m} c^2 \sqrt{1 + 2\Phi/c^2}$; for $|\Phi| \ll c^2$ it reduces to the linear approximation above.

Domain: Weak field ($|\Phi| \ll c^2$), static or slowly varying, test particle.

Confirmed by: Pound-Rebka (1959), GPS (daily), Hafele-Keating (1971), LIGO/Virgo (2015-present).

4. Extension 2: Electromagnetism (Maxwell-Einstein, 1905-1915)

For a particle with charge q and rest mass m_0 in an electromagnetic field described by scalar potential ϕ and vector potential \mathbf{A} , the total energy is:

$$\mathbf{E} = \sqrt{(\mathbf{p} - q\mathbf{A})^2 c^2 + m_0^2 c^4} + q\phi$$

For a weak, static electric field ($\mathbf{A} = \mathbf{0}$, $q\phi \ll m_0 c^2$):

$$\mathbf{E} \approx \gamma m_0 c^2 + q\phi$$

Minor clarification: γ in the approximation uses the kinetic momentum $\mathbf{p}_{\text{kin}} = \mathbf{p} - q\mathbf{A}$; for $\mathbf{A} = \mathbf{0}$ both coincide.

Domain: Exact for all classical EM fields; approximation valid for weak electrostatic fields.

Confirmed by: Maxwell's equations (all electronics), QED, particle accelerators.

5. Extension 3: Higgs Mechanism (Higgs, 1964 - confirmed 2012)

For elementary fermions, the rest mass is given by the Higgs vacuum expectation value $v \approx 246 \text{ GeV}$ and Yukawa coupling y :

$$m c^2 = y \cdot v$$

For the W and Z bosons: $m_W c^2 = \frac{1}{2} g v$, $m_Z c^2 = \frac{1}{2} \sqrt{g^2 + g'^2} v$.

Minor clarification: Unlike gravity and electromagnetism, the Higgs field does *not* add energy to a pre-existing mass term. It **generates** the mass itself. This is a **conceptual** extension.

Confirmed by: W/Z masses (1983), top quark mass (1995), Higgs boson discovery (2012, ATLAS/CMS), Yukawa coupling measurements.

6. Summary of the Three Extensions

Gravity - Valid

Approx: $\mathbf{E} \approx \gamma(m c^2 + m\Phi)$

Confirmed by: GPS, Pound-Rebka, LIGO

Electromagnetism - Valid

Exact: $\mathbf{E} = \sqrt{(\mathbf{p} - q\mathbf{A})^2 c^2 + m_0^2 c^4} + q\phi$

Weak-field: $\mathbf{E} \approx \gamma \mathbf{m}_0 \mathbf{c}^2 + \mathbf{q}\phi$

Confirmed by: all electrodynamics, QED, accelerators

Higgs - Conceptual

Formula: $\mathbf{m}\mathbf{c}^2 = \mathbf{y} \cdot \mathbf{v}$ (mass generation)

Confirmed by: LHC 2012, W/Z masses, top quark

Key observation: Gravity and EM add energy to pre-existing $\mathbf{m}_0 \mathbf{c}^2$; Higgs generates $\mathbf{m}_0 \mathbf{c}^2$ itself. No single formula unifies all three. The strong interaction (QCD) does **not** admit a simple additive extension.

7. Open Questions for a Future Extended Energy Equation

How does the strong interaction (QCD confinement, gluon field energy) enter the energy equation in a compact form?

Can gravity and electromagnetism be unified into a single covariant extension beyond the linear approximation?

What is the correct energy equation in quantum gravity?

How do non-linear field contributions modify the additive structure of the energy equation?

Can the conceptual Higgs mechanism be reformulated as a structural additive term?

8. Conclusion

What have we learned? First, $\mathbf{E} = \mathbf{m}\mathbf{c}^2$ is not the final word when fields are present. Second, different fields require different mathematical structures. Third, the Higgs mechanism shows that even "rest mass" is field-derived. These three known extensions are not contradictions of Einstein - they are consequences of taking his equation seriously in a world filled with fields.

This paper provides the conceptual basis upon which I am developing a new extension of the energy equation. It is also intended to encourage other physicists to think further. Building on this foundation, the next step will be to propose a unified extension that combines the additive structure of gravity and electromagnetism with the generative insight of the Higgs mechanism, while addressing the open questions listed above.

The door is open.

Acknowledgments

I thank my mother for teaching me patience, the forefathers of this revolutionary equation, and I acknowledge the intellectual tradition that connects physics and spirituality. Thanks to Allah for the love to research.

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